① a) $\int \frac{e^{2x}dx}{\sqrt{1+e^x}} = \int \frac{e^x e^x dx}{\sqrt{1+e^x}} =$ $\quad t^2 = 1+e^x \Rightarrow e^x = t^2-1$

$\quad 2t\,dt = e^x dx$

$= \int \frac{(t^2-1)\,2t\,dt}{t} = 2\int (t^2-1)\,dt = 2\frac{t^3}{3} - 2t + K = \boxed{\frac{2\sqrt{(1+e^x)^3}}{3} - 2\sqrt{1+e^x} + K}$

b) $\int \frac{\operatorname{sen} 2x}{3+\operatorname{sen}^2 x}dx = \int \frac{2\operatorname{sen} x \cos x}{3+\operatorname{sen}^2 x}dx =$ El numerador es la derivada del denominador

$= \boxed{\ln|3+\operatorname{sen}^2 x| + K}$

c) $\int \frac{1}{x^2+2x+3}dx =$ $\quad x^2+2x+3=0 \quad x = \cancel{\frac{-2\pm\sqrt{4-12}}{}}$ raíces complejas

$\sim (x+1)^2+2 = x^2+2x+1+2$

$= \int \frac{1}{(x+1)^2+2}dx = \int \frac{\frac{1}{2}}{\frac{(x+1)^2}{2}+1}dx = \frac{1}{2}\int \frac{dx}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1} =$

$t = \frac{x+1}{\sqrt{2}} \quad dt = \frac{dx}{\sqrt{2}} \qquad = \frac{1}{2}\int \frac{\sqrt{2}\,dt}{t^2+1} = \frac{\sqrt{2}}{2}\operatorname{arctg} t + K =$

$= \boxed{\frac{\sqrt{2}}{2}\operatorname{arctg}\frac{x+1}{\sqrt{2}} + K}$

d) $\int \frac{1}{4x^2-9}dx \qquad \frac{1}{4x^2-9} = \frac{A}{2x+3} + \frac{B}{2x-3} = \frac{A(2x-3)+B(2x+3)}{4x^2-9}$

$1 = A(2x-3)+B(2x+3)$

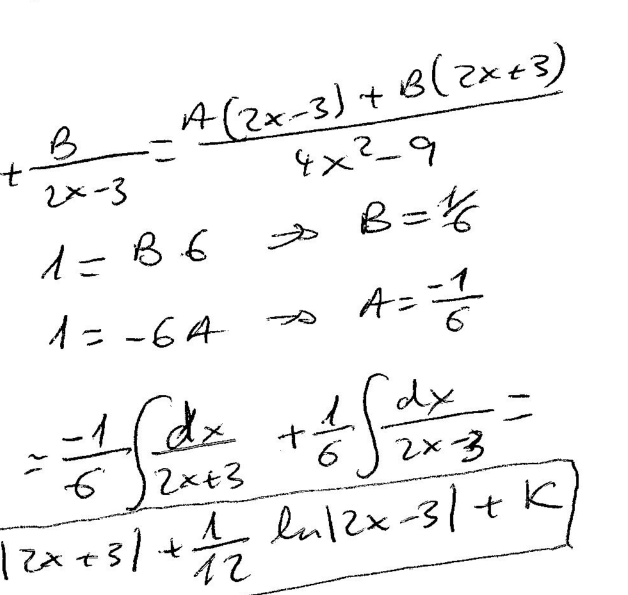
$x = \frac{3}{2} \Rightarrow 1 = B\,6 \Rightarrow B = \frac{1}{6}$

$x = \frac{-3}{2} \Rightarrow 1 = -6A \Rightarrow A = \frac{-1}{6}$

$\int \frac{1}{4x^2-9}dx = \int \frac{-1/6}{2x+3}dx + \int \frac{1/6}{2x-3}dx = \frac{-1}{6}\int \frac{dx}{2x+3} + \frac{1}{6}\int \frac{dx}{2x-3} =$

$= \frac{-1}{12}\int \frac{2dx}{2x+3} + \frac{1}{12}\int \frac{2dx}{2x-3} = \boxed{\frac{-1}{12}\ln|2x+3| + \frac{1}{12}\ln|2x-3| + K}$

③ Área comprendida entre $y=|x^2-9|$, el eje de abscisas y las rectas $x=1$ e $x=4$



$y = |x^2-9| = \begin{cases} x^2-9 & \text{si } x \geq 3 \text{ ó } x \leq -3 \\ -x^2+9 & \text{si } x \in (-3,3) \end{cases}$

Área $= \int_1^3 (-x^2+9)dx + \int_3^4 (x^2-9)dx =$

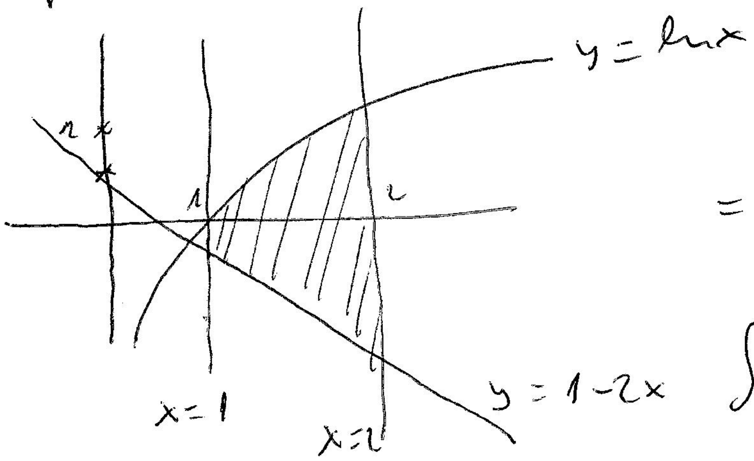
$= \left[\frac{-x^3}{3}+9x\right]_1^3 + \left[\frac{x^3}{3}-9x\right]_3^4 = \frac{-27}{3}+27-\left(\frac{-1}{3}+9\right) + \frac{64}{3} - 36 - \left(\frac{27}{3}-27\right) = \boxed{\frac{38}{3}u^2}$

④ Sea $f(x)=\int_2^x \frac{1}{1+\ln t}\,dt$. Calcular $\lim_{x\to 2} \frac{f(x)}{x-2}$

$$\lim_{x\to 2} \frac{f(x)}{x-2} = \lim_{x\to 2} \frac{\int_2^x \frac{1}{1+\ln t}\,dt}{x-2} = \frac{\int_2^2 \frac{1}{1+\ln t}\,dt}{2-2} = \left(\frac{0}{0}\right) \text{ind} =$$

$$\underset{\text{L'Hôpital}}{=} \lim_{x\to 2} \frac{\frac{1}{1+\ln x}}{1} = \boxed{\frac{1}{1+\ln 2}}$$

⑤ Dadas $f(x)=\ln x$ y $g(x)=1-2x$. Hallar el área del recinto plano limitado por las rectas $x=1$, $x=2$ y las gráficas de $f(x)$ y $g(x)$



$$\text{Área} = \int_1^2 (\ln x - 1 + 2x)\,dx =$$

$$= \int_1^2 \ln x\,dx + \left(-x + x^2\right)\Big|_1^2$$

$$\int \ln x\,dx = \begin{bmatrix} u = \ln x & du = \frac{dx}{x} \\ dv = dx & v = x \end{bmatrix} =$$

$$= x\ln x - \int x\frac{dx}{x} = x\ln x - x$$

$$\text{Área} = \Big[x\ln x - x - x + x^2\Big]_1^2 = 2\ln 2 - 2 - 2 + 4 - (1\ln 1 - 1 - 1 + 1) =$$

$$= \boxed{2\ln 2 + 1 \quad u^2}$$